Similarity-score-based likelihood ratios do not take account of typicality

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- All opinions expressed are those of the presenter and, unless explicitly stated otherwise, should not be construed as representing the policies or positions of any organizations with which the presenters are associated.

Slides

- https://geoff-morrison.net/#ICFIS2023

Specific-Source Likelihood Ratio

Specific-source likelihood ratio

- What is the likelihood of obtaining the measured properties of the item of questioned source if it came from the specific known source?

divided by

- What is the likelihood of obtaining the measured properties of the item of questioned source if it came not from the specific known source but from some other source selected at random from the relevant population?

Specific-source likelihood ratio

$$\Lambda = \frac{f\left(x_\mathrm{q} \middle| M_\mathrm{k}\right)}{f\left(x_\mathrm{q} \middle| M_\mathrm{r}\right)}$$

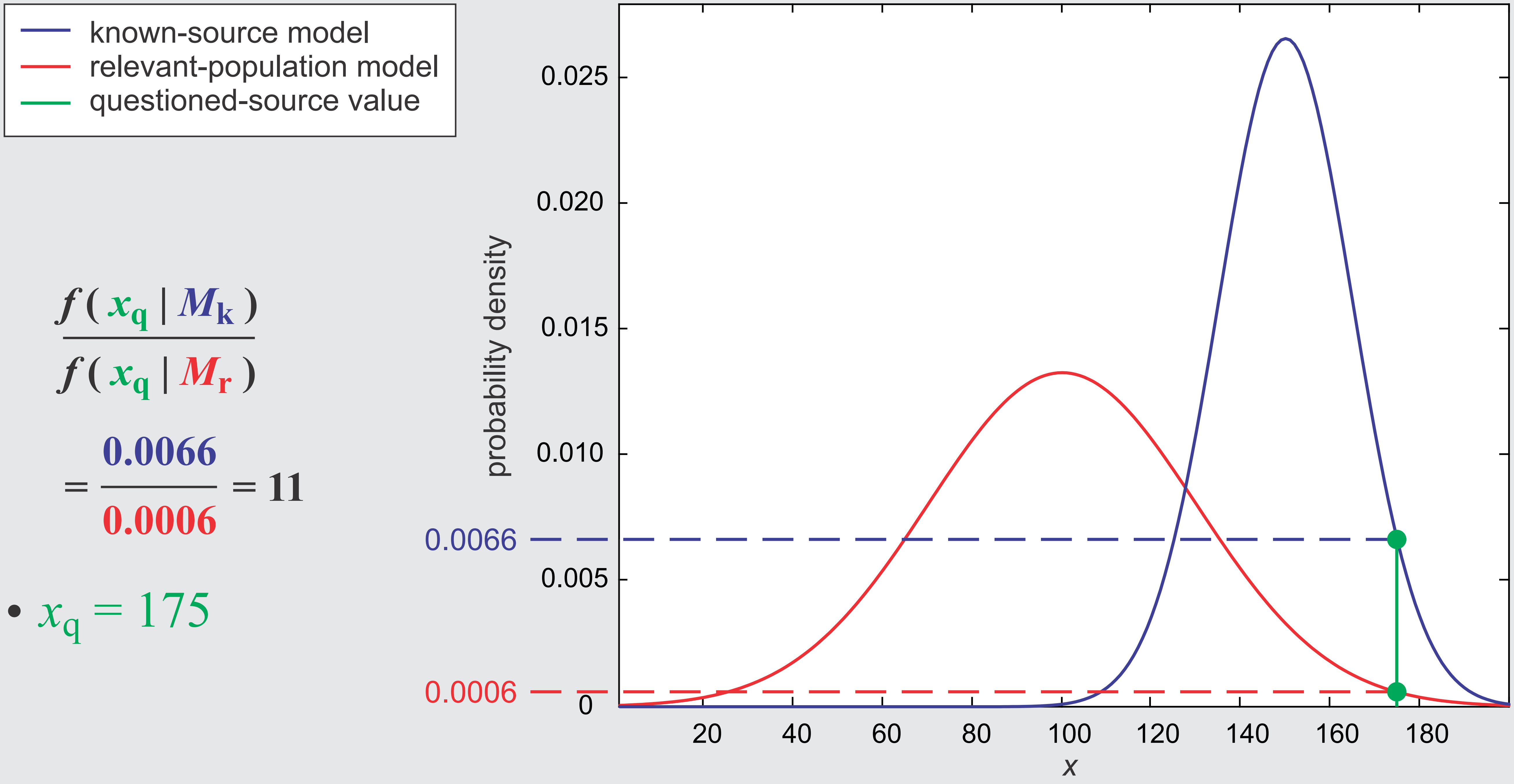
$$\Lambda = \frac{f\left(x_\mathrm{q} \middle| \mu_\mathrm{k}, \sigma_\mathrm{k}^2\right)}{f\left(x_\mathrm{q} \middle| \mu_\mathrm{r}, \sigma_\mathrm{r}^2\right)} = \frac{f\left(x_\mathrm{q} \middle| \mu_\mathrm{k}, \sigma_\mathrm{w}^2\right)}{f\left(x_\mathrm{q} \middle| \mu_\mathrm{r}, \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2\right)}$$

Specific-source likelihood ratio

$$\frac{f(x_q \mid M_k)}{f(x_q \mid M_r)}$$

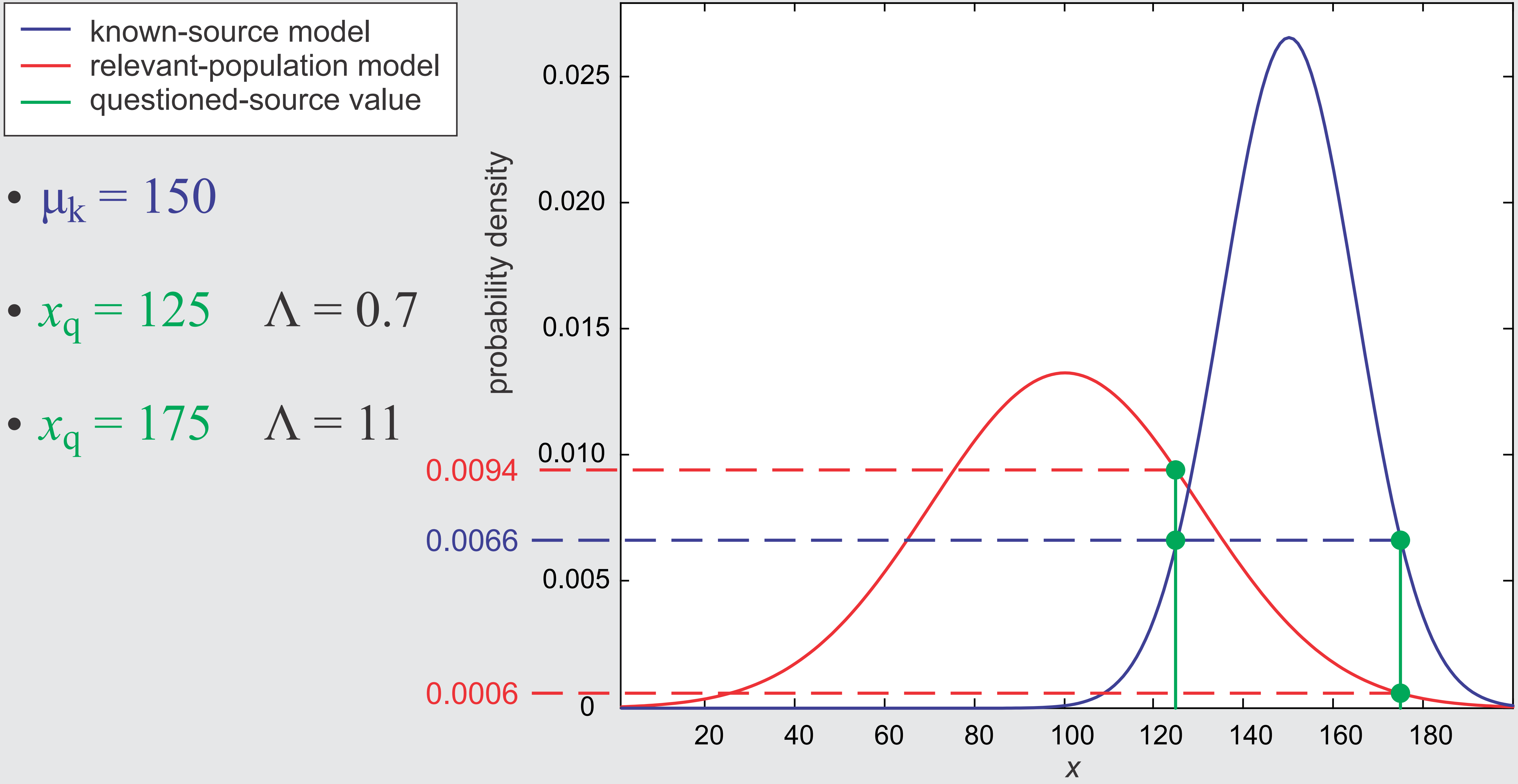
$$= \frac{0.0066}{0.0006} = 11$$

- $x_q = 175$



Specific-source likelihood ratios – same degree of similarity

- $\mu_k = 150$
- $x_q = 125 \quad \Lambda = 0.7$
- $x_q = 175 \quad \Lambda = 11$



Common-Source Likelihood Ratio

Common-source likelihood ratio

- What is the likelihood of obtaining the measured properties of the items of questioned and known source if they both came from the same source (a source selected at random from the relevant population)?

  divided by

- What is the likelihood of obtaining the measured properties of the items of questioned and known source if they each came from a different source (each a source selected at random from the relevant population)?

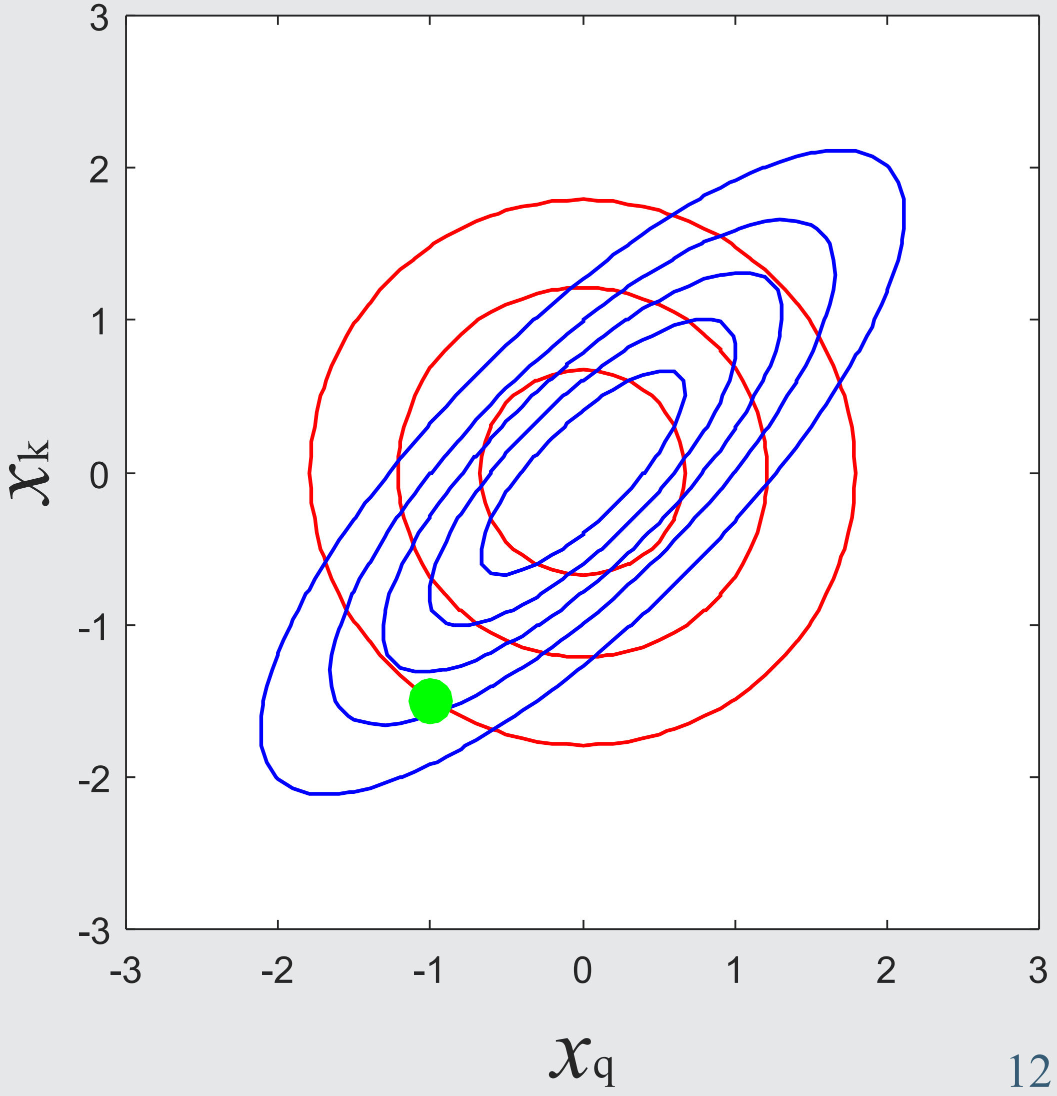
Common-source likelihood ratio

$$\Lambda = \frac{f\left(x_\mathrm{q}, x_\mathrm{k} \middle| M_\mathrm{s}\right)}{f\left(x_\mathrm{q} \middle| M_\mathrm{d}\right) f\left(x_\mathrm{k} \middle| M_\mathrm{d}\right)}$$

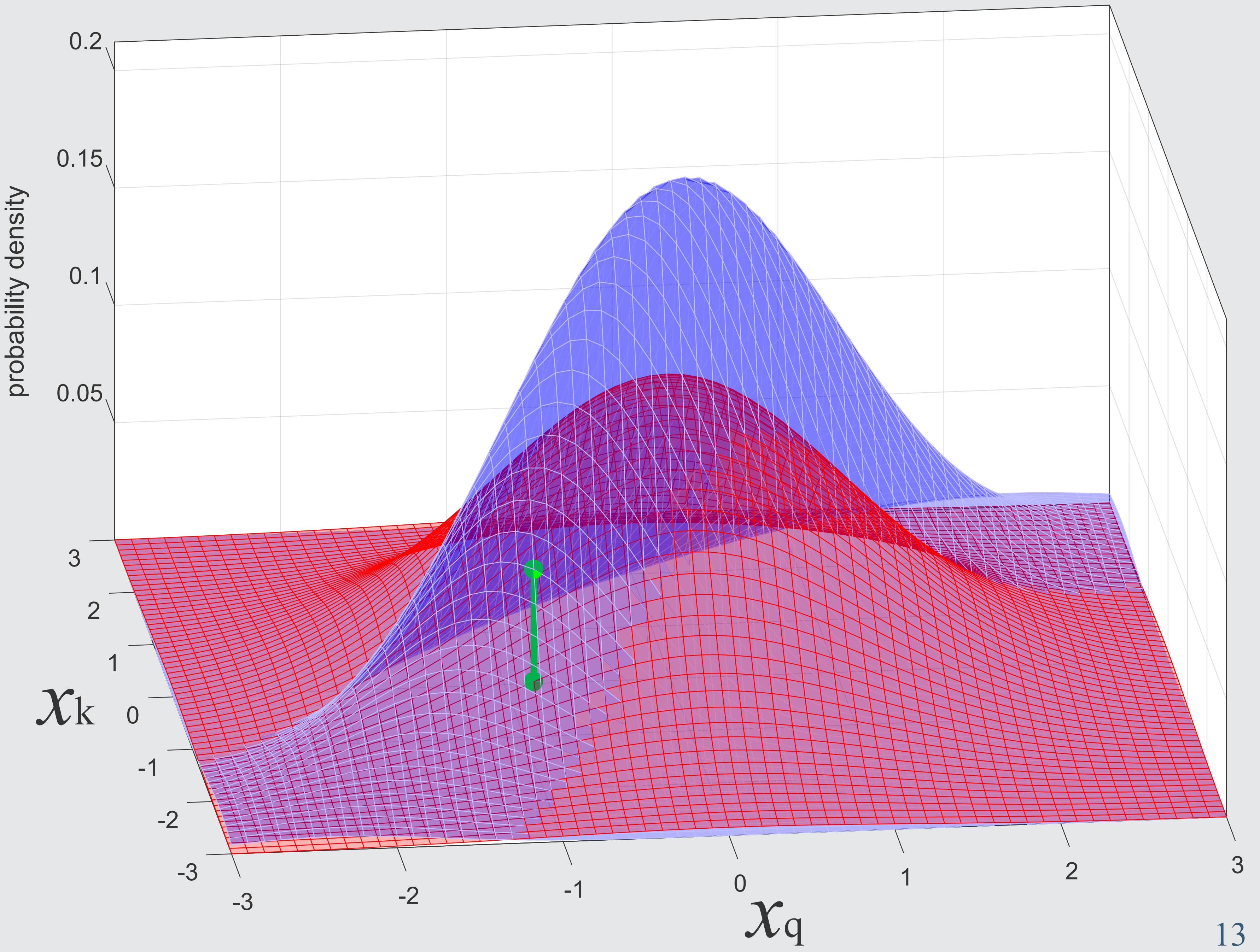
$$\Lambda = \frac{f\left(\begin{bmatrix} x_\mathrm{q} \\ x_\mathrm{k} \end{bmatrix} \middle| \begin{bmatrix} \mu_\mathrm{r} \\ \mu_\mathrm{r} \end{bmatrix}, \begin{bmatrix} \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 & \sigma_\mathrm{b}^2 \\ \sigma_\mathrm{b}^2 & \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 \end{bmatrix}\right)}{f\left(\begin{bmatrix} x_\mathrm{q} \\ x_\mathrm{k} \end{bmatrix} \middle| \begin{bmatrix} \mu_\mathrm{r} \\ \mu_\mathrm{r} \end{bmatrix}, \begin{bmatrix} \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 & 0 \\ 0 & \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 \end{bmatrix}\right)}$$

Common-source likelihood ratio

$$\Lambda = \frac{f\left(\begin{bmatrix} x_\mathrm{q} \\ x_\mathrm{k} \end{bmatrix} \middle| \begin{bmatrix} \mu_\mathrm{r} \\ \mu_\mathrm{r} \end{bmatrix}, \begin{bmatrix} \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 & \sigma_\mathrm{b}^2 \\ \sigma_\mathrm{b}^2 & \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 \end{bmatrix}\right)}{f\left(\begin{bmatrix} x_\mathrm{q} \\ x_\mathrm{k} \end{bmatrix} \middle| \begin{bmatrix} \mu_\mathrm{r} \\ \mu_\mathrm{r} \end{bmatrix}, \begin{bmatrix} \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 & 0 \\ 0 & \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 \end{bmatrix}\right)}$$

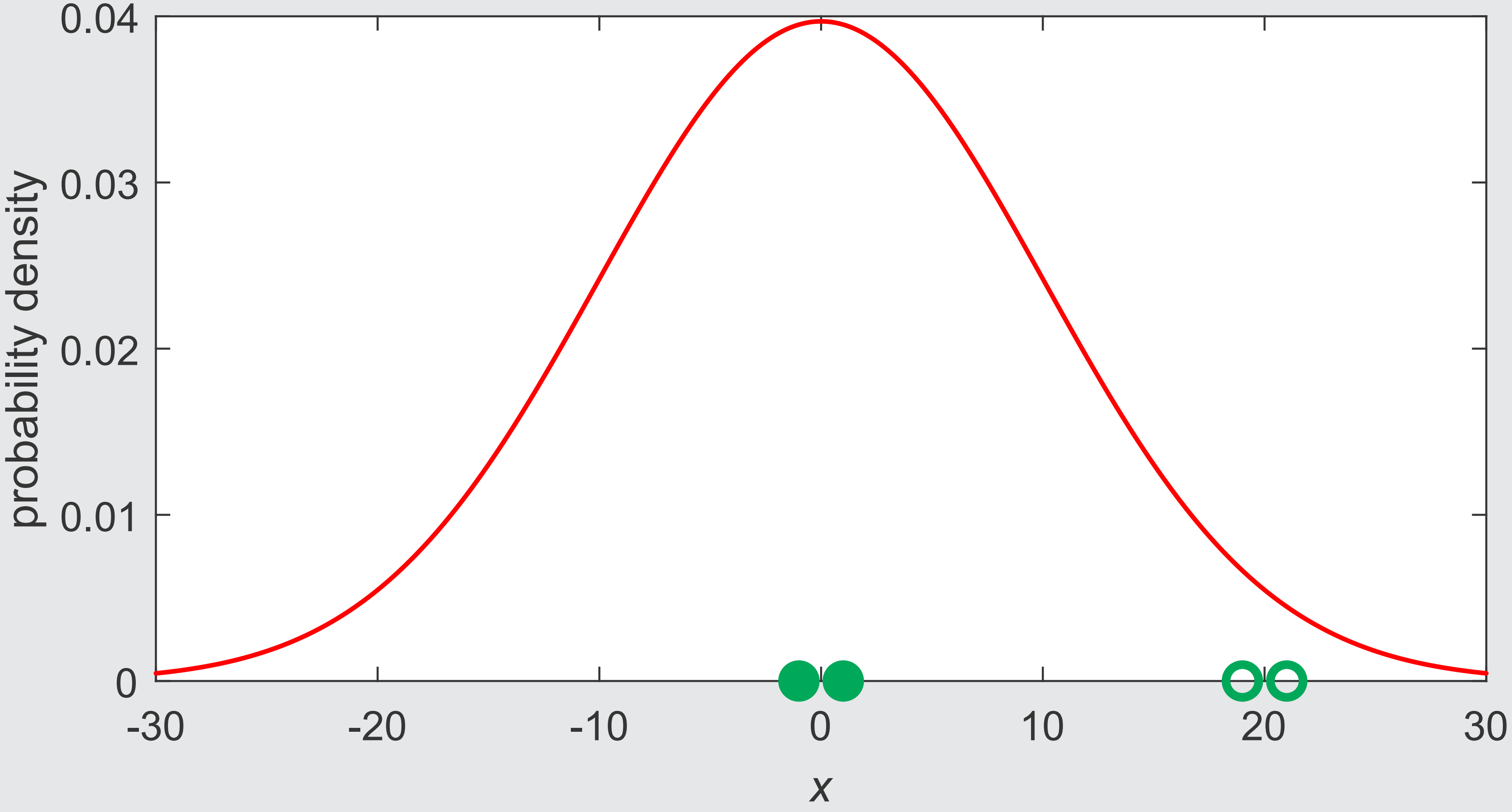


Common-source likelihood ratio



Common-source likelihood ratios – same degree of similarity

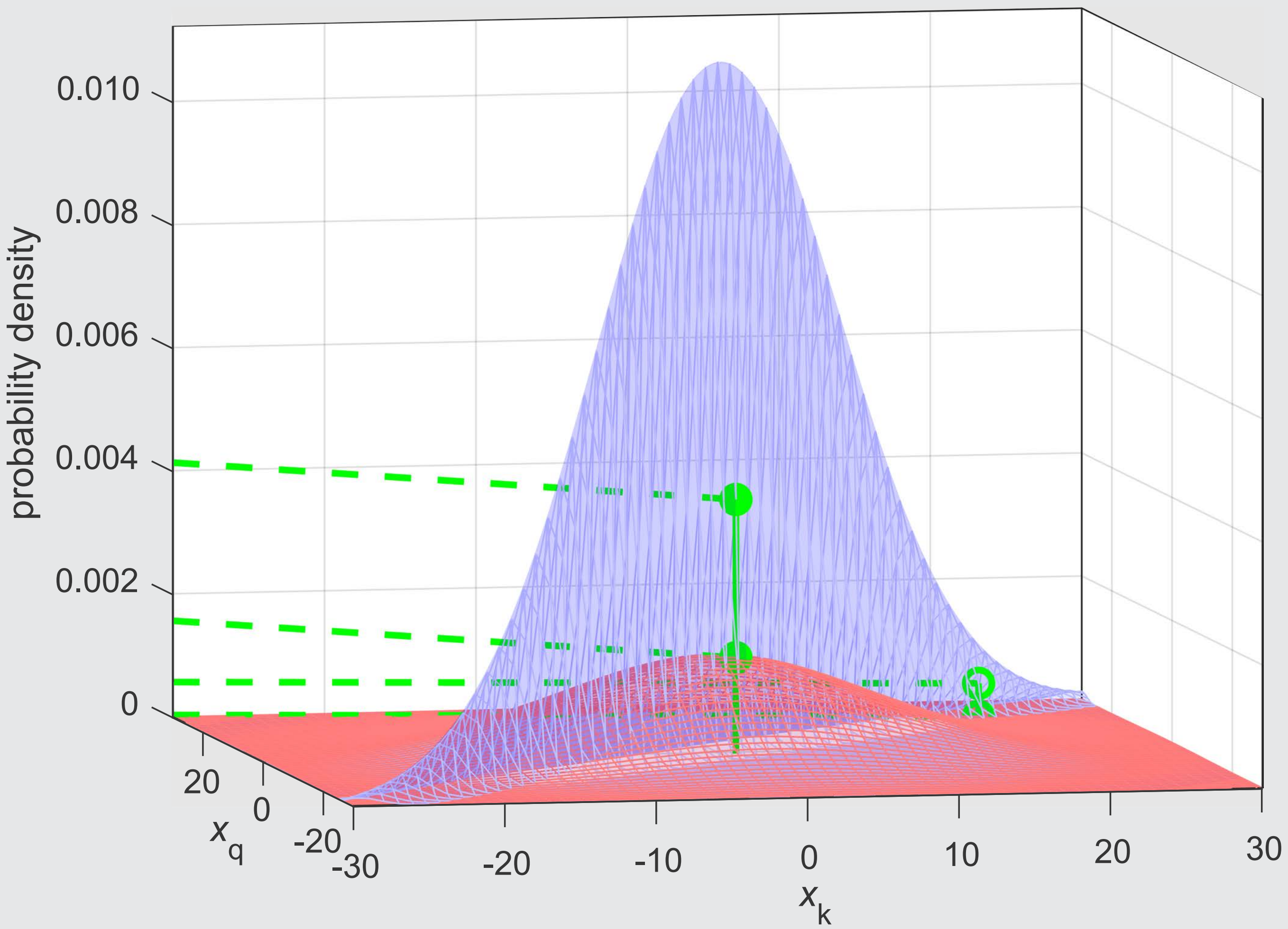
- $\mu_r = 0$
- $\sigma_b^2 = 100$
- $\sigma_w^2 = 1$



- $x_q = -1 \quad x_k = 1$
- $x_q = 19 \quad x_k = 21$

Common-source likelihood ratios – same degree of similarity

$$\Lambda = \frac{f\left(\begin{bmatrix} x_\mathrm{q} \\ x_\mathrm{k} \end{bmatrix} \middle| \begin{bmatrix} \mu_\mathrm{r} \\ \mu_\mathrm{r} \end{bmatrix}, \begin{bmatrix} \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 & \sigma_\mathrm{b}^2 \\ \sigma_\mathrm{b}^2 & \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 \end{bmatrix}\right)}{f\left(\begin{bmatrix} x_\mathrm{q} \\ x_\mathrm{k} \end{bmatrix} \middle| \begin{bmatrix} \mu_\mathrm{r} \\ \mu_\mathrm{r} \end{bmatrix}, \begin{bmatrix} \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 & 0 \\ 0 & \sigma_\mathrm{w}^2 + \sigma_\mathrm{b}^2 \end{bmatrix}\right)}$$



- $x_\mathrm{q} = -1 \quad x_\mathrm{k} = 1 \qquad \Lambda = (41 \times 10^{-4}) / (16 \times 10^{-4}) = 2.6$
- $x_\mathrm{q} = 19 \quad x_\mathrm{k} = 21 \qquad \Lambda = (56 \times 10^{-5}) / (3.0 \times 10^{-5}) = 19$

Similarity-Score-Based Likelihood Ratio

Similarity-score-based likelihood ratio

- What is the likelihood of obtaining the measured degree of similarity between the items of questioned and known source if they both came from the same source (a source selected at random from the relevant population)?

  divided by

- What is the likelihood of obtaining the measured degree of similarity between the items of questioned and known source if they each came from a different source (each a source selected at random from the relevant population)?

Similarity-score-based likelihood ratio

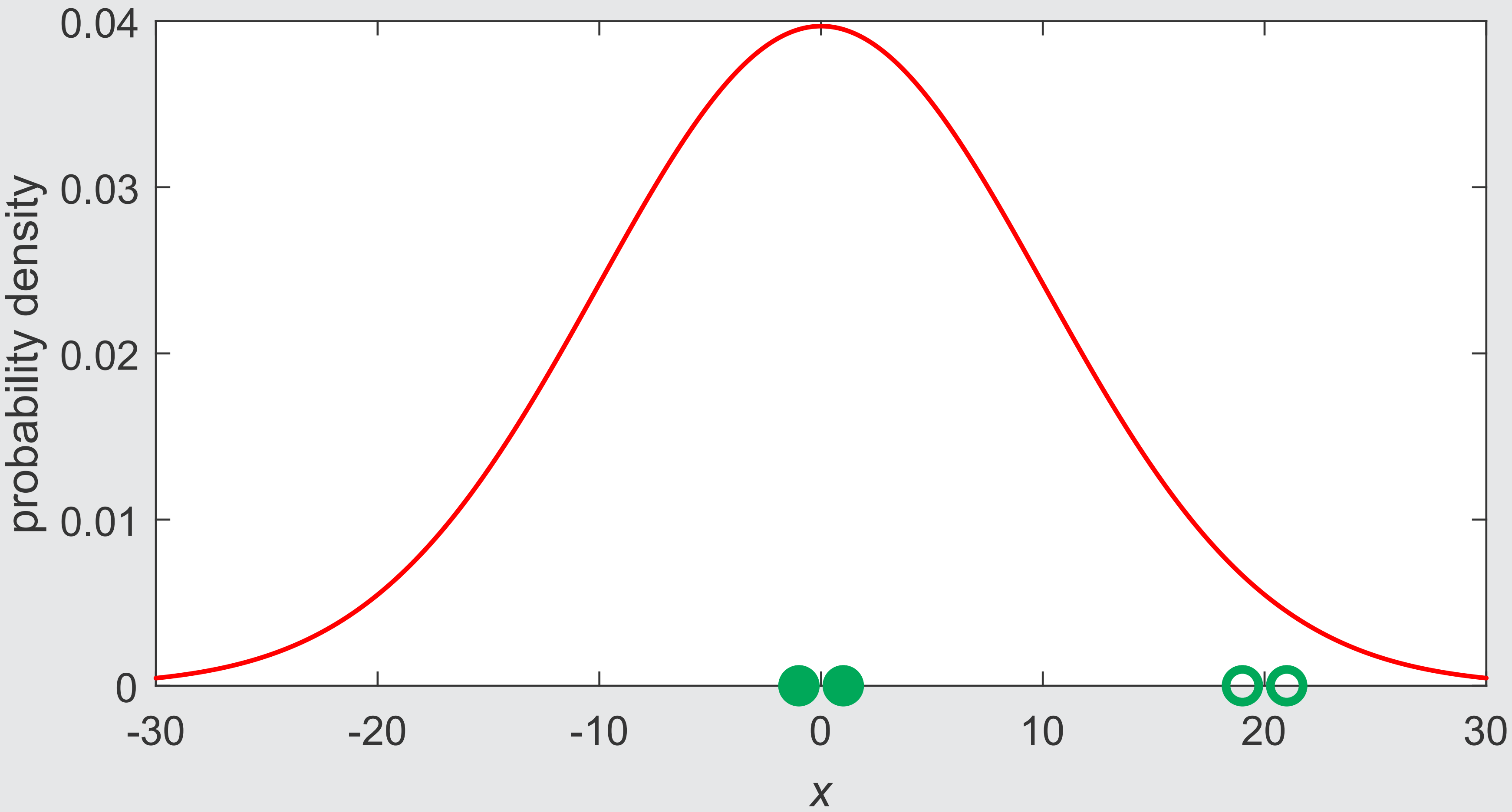
$$\Lambda = \frac{f\left(\delta\left(x_{\mathrm{q}}, x_{\mathrm{k}}\right) \middle| M_{\delta,\mathrm{s}}\right)}{f\left(\delta\left(x_{\mathrm{q}}, x_{\mathrm{k}}\right) \middle| M_{\delta,\mathrm{d}}\right)}$$

$$\delta\left(x_{\mathrm{q}}, x_{\mathrm{k}}\right) = \left|x_{\mathrm{q}} - x_{\mathrm{k}}\right|$$

- degree of similarity is the inverse of degree of difference

Similarity-score-based likelihood ratios – same degree of similarity

- $\mu_r = 0$
- $\sigma_b^2 = 100$
- $\sigma_w^2 = 1$



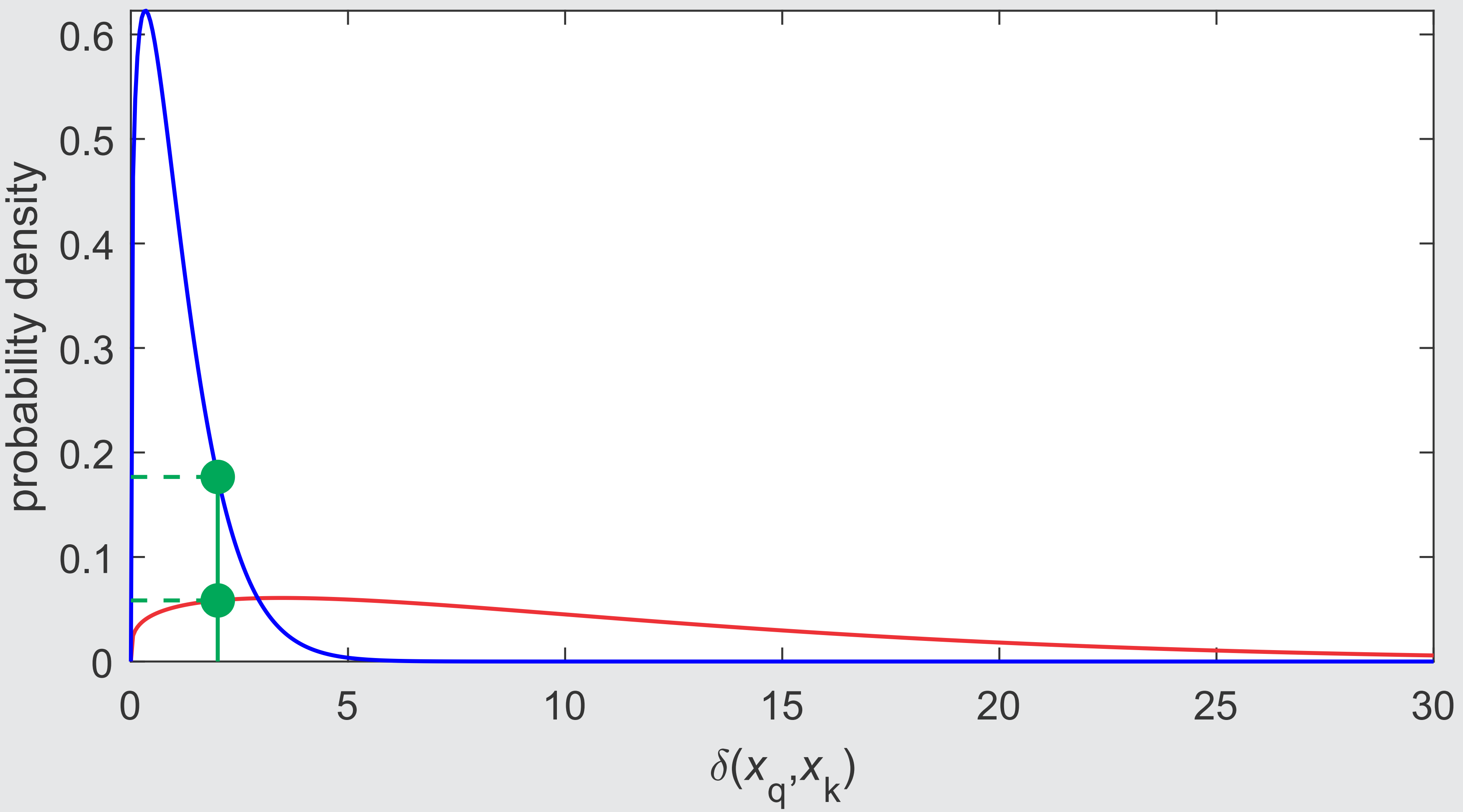
- $x_q = -1 \quad x_k = 1$
- $x_q = 19 \quad x_k = 21$

Similarity-score-based likelihood ratios – same degree of similarity

$$\Lambda = \frac{f\left(\delta\left(x_\mathrm{q}, x_\mathrm{k}\right) \middle| M_{\delta,\mathrm{s}}\right)}{f\left(\delta\left(x_\mathrm{q}, x_\mathrm{k}\right) \middle| M_{\delta,\mathrm{d}}\right)}$$

$$\delta\left(x_\mathrm{q}, x_\mathrm{k}\right) = \left|x_\mathrm{q} - x_\mathrm{k}\right|$$

- Weibull distributions



- $x_\mathrm{q} = -1$ $x_\mathrm{k} = 1$ $\delta(-1, 1) = 2$ $\Lambda = 0.18 / 0.058 = 3.1$
- $x_\mathrm{q} = 19$ $x_\mathrm{k} = 21$ $\delta(19, 21) = 2$ $\Lambda = 0.18 / 0.058 = 3.1$

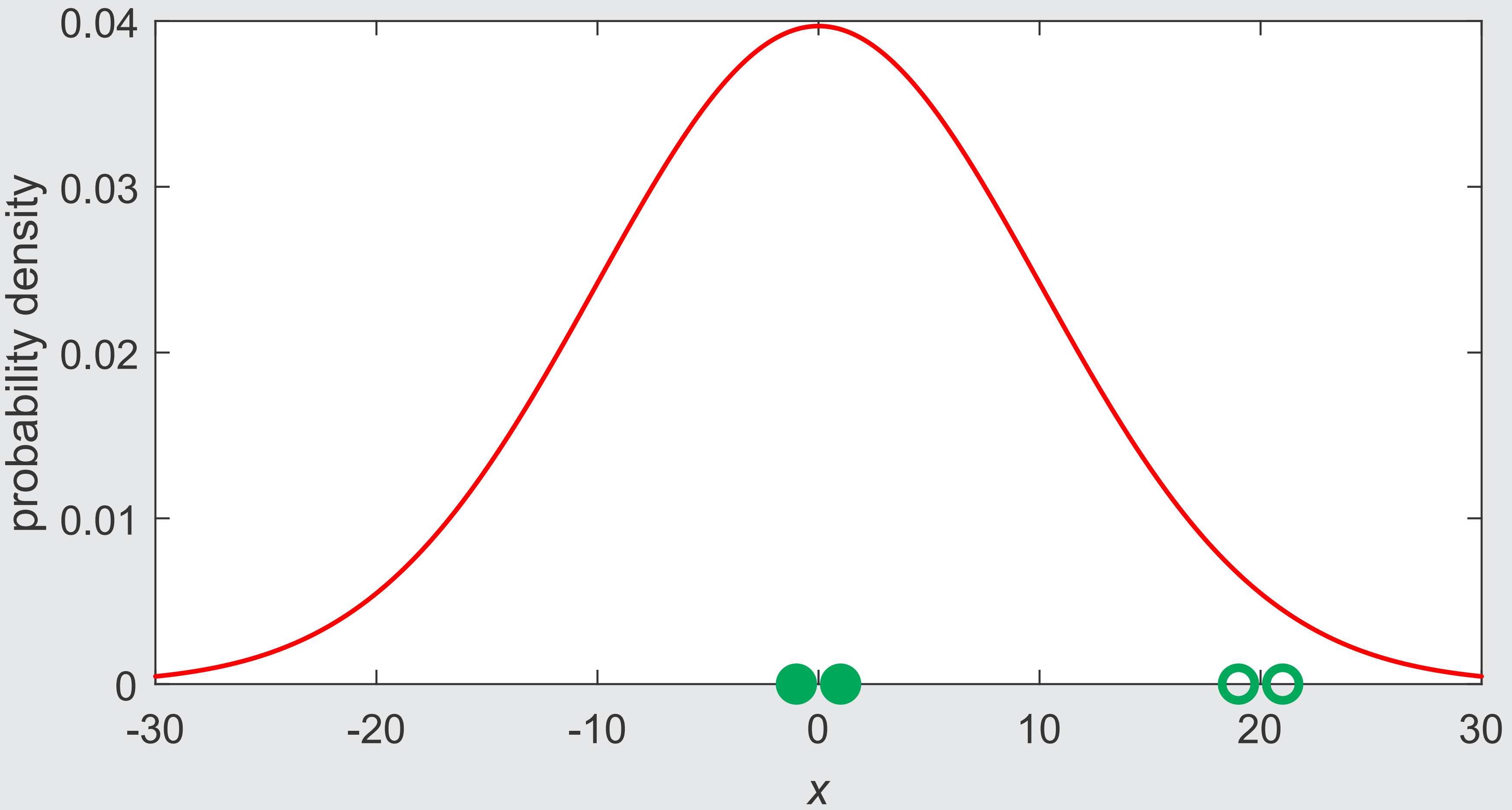
Common-Source Likelihood Ratio

v

Similarity-Score-Based Likelihood Ratio

Common-source versus similarity-score-based likelihood ratios

- $\mu_r = 0$
- $\sigma_b^2 = 100$
- $\sigma_w^2 = 1$



- $x_q = -1$ $x_k = 1$ $\Lambda_{\text{common-source}} = 2.6$ $\Lambda_{\text{similarity-score}} = 3.1$
- $x_q = 19$ $x_k = 21$ $\Lambda_{\text{common-source}} = 19$ $\Lambda_{\text{similarity-score}} = 3.1$

Conclusion and Comments

Conclusion

- Similarity-score-based likelihood ratios do not properly take account of typicality.

- Similarity-score-based likelihood ratios should not be used.

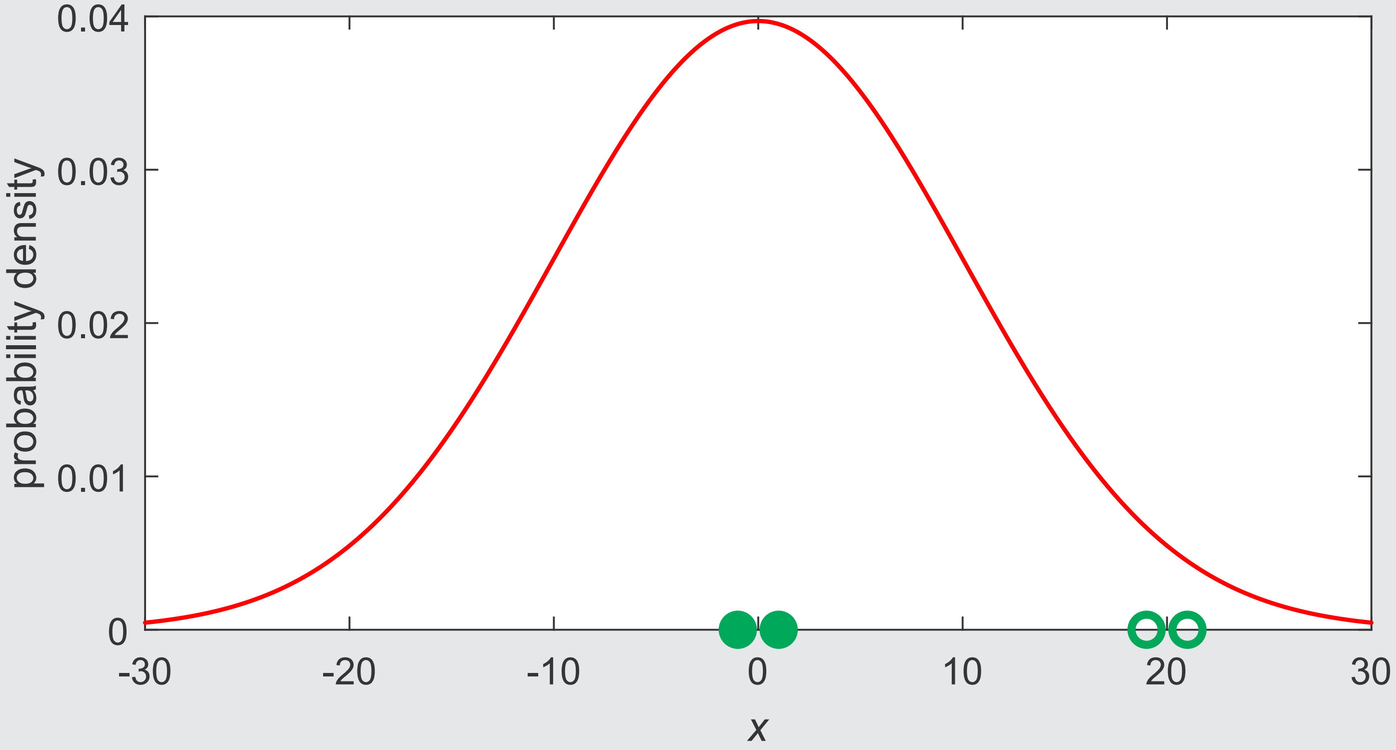
- Common-source likelihood ratios should be used instead.

Comments

- Similarity scores are not a legitimate alternative “evidence” to features.

- Similarity scores are derived from features.

- The features are the “evidence”.



Comments

- Calculating likelihood ratios from similarity scores is not the same as calibration.
- Calibration converts uncalibrated likelihood ratios to calibrated likelihood ratios.
- Logarithms of uncalibrated likelihood ratios are referred to as “scores”, but these scores take account of both similarity and typicality.

Comments

- Using simulations, Morrison & Enzinger (2018) compared similarity-score-based models with specific-source models. The comparison should have been with common-source models.

- Using real data, Morrison & Enzinger (2018) demonstrated that methods that take account of both similarity and typicality outperform methods that only take account of similarity.

Morrison G.S., Enzinger E. (2018). **Score based procedures for the calculation of forensic likelihood ratios – Scores should take account of both similarity and typicality**. *Science & Justice*, 58, 47–58. https://doi.org/10.1016/j.scijus.2017.06.005

Comments

- For almost two decades, automatic speaker recognition and forensic voice comparison have successfully applied common-source models to high-dimensional data.

Morrison G.S., Enzinger E., Ramos D., González-Rodríguez J., Lozano-Díez A. (2020). **Statistical models in forensic voice comparison**. In Banks D.L., Kafadar K., Kaye D.H., Tackett M. (Eds.), *Handbook of Forensic Statistics* (Ch. 20, pp. 451–497). Boca Raton, FL: CRC. https://doi.org/10.1201/9780367527709
Preprint at https://forensic-voice-comparison.net/handbook-of-forensic-statistics/

Morrison G.S., Weber P., Enzinger E., Labrador B., Lozano-Díez A., Ramos D., González-Rodríguez J. (2023). **Forensic voice comparison: Human-supervised-automatic approach**. In Houck M., Wilson L., Eldridge H., Lewis S., Lothridge K., Reedy P. (Eds.), *Encyclopedia of Forensic Sciences* (3rd Ed.), vol. 2, pp. 720–736. Elsevier. https://doi.org/10.1016/B978-0-12-823677-2.00182-3
Preprint at https://forensic-voice-comparison.net/encyclopedia/

Thank You