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Incorrect formula for calculation of likelihood ratios used in forensic anthropology: Comments on Scott & Rogers (2026)

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Abstract

Scott & Rogers (2026) https://doi.org/10.1016/j.forsciint.2025.112673 promotes the use of the likelihood-ratio framework in forensic anthropology. This is welcome. Unfortunately, Scott & Rogers (2026) uses an incorrect formula for the calculation of likelihood ratios. This incorrect formula did not originate in Scott & Rogers (2026). It has, for some time, been used in the forensic anthropology literature; an earlier occurrence appears in Steadman et al. (2006) https://doi.org/10.1002/ajpa.20393. Scott & Rogers (2026) also uses confusing language and mathematical notation that are non-standard compared to the norms of the forensic-inference-and-statistics literature. This letter to the editor is offered in the hope that it will help prevent repetition of these problems.

Keywords

Calculation; Forensic anthropology; Formula; Likelihood ratio; Vocabulary

Letter to the Editor:

Scott & Rogers [1] illustrates calculation of likelihood ratios in forensic anthropology, particularly calculation of likelihood ratios based on observations made on skeletal remains. Forensic anthropology appears to be a branch of forensic science in which there has so far been little adoption of the likelihood-ratio framework. [1]'s promotion of the likelihood-ratio framework in forensic anthropology is therefore welcome.

In §3.3 of [1], a likelihood ratio of 107.25 is calculated. That likelihood ratio is calculated using a correct formula for the calculation of likelihood ratios. In §3.2 of [1], a likelihood ratio of 1.6 is calculated using Eq. 4 of [1]. Unfortunately, Eq. 4 of [1] is not a correct formula for the calculation of likelihood ratios. Unfortunately, the presentation in [1] is also confusing because it uses language and mathematical notation that are inexact and are non-standard compared to the norms of the forensic-inference-and-statistics literature. The present letter to the editor is offered in the hope that it will help prevent repetition of these problems.

Using standard mathematical notation, a correct formula for the calculation of a likelihood ratio, Λ, is given in Equation (1).

$$(1) \quad \Lambda = \frac{p(E|H_1)}{p(E|H_2)}$$

In Equation (1), H_1 and H_2 represent mutually exclusive hypotheses (aka propositions). [1] uses two pairs of hypotheses. Interpreting and translating those pairs of hypotheses into more standard and more exact language, one pair is:

- H_1: The skeletal remains are those of Person A.
- H_2: The skeletal remains are not those of Person A, but are those of some other person selected at random from the relevant population.

and the other pair is:

- H_1: The skeletal remains are those of an adult biological male human from the relevant population.
- H_2: The skeletal remains are those of an adult biological female human from the relevant population.

In [1], both pairs of hypotheses are usually written as "correct identification" and "incorrect identification". These are inexact and potentially confusing. One reason they are potentially confusing is because "identification" is usually used as a categorical conclusion. Even without the confusing vocabulary choice, the inexactitude is a problem: Hypotheses specify the question that the likelihood ratio answers. It is therefore essential for a forensic practitioner to clearly communicate the exact hypotheses that they have adopted. Likewise, it is important for a research paper to clearly communicate the exact hypotheses being discussed.

What constitutes the relevant population is part of the exact definition of the question. In [1], the terms "population at large" and "identification universe" are used. These non-standard terms are vague and suggest something broader than what is meant by relevant population.[1] For the first pair of hypotheses (Person A versus not Person A), the relevant population is a specific population from which, in the specific case, the skeletal remains could have originated had they not originated from Person A. [1] discusses adult persons missing in Canada as a potential relevant population, and for the purpose of illustrating calculation of likelihood ratios, [1] uses "individuals of European affinity" as the relevant population.

In Equation (1), E (the evidence) represents observations made on the items of interest. Observations describe properties of items of interest. Observations can be the result of instrumental measurement or of human perception. Observations can be quantitative, either continuously valued or discretely valued, or can be categorical, e.g., some property is present or is absent.[2]

In [1], for the illustration of the calculation of likelihood ratios given the first pair of hypotheses stated above (Person A versus not Person A), a categorical observation is

[1] A general discussion of the concept of relevant population is provided in [2] ch. 3.

[2] See [3] for illustrations of the calculation of likelihood ratios using continuously-valued quantitative measurements made on skeletal remains.

used:

- E = absence of a transverse process on the first lumbar vertebra (L1 vertebra) of the recovered remains

Antemortem medical records for Person A indicated that they lacked a transverse process on L1. Therefore, the probability that the recovered L1 vertebra would lack a transverse process if H_1 were true, i.e., if it were the L1 vertebra of Person A, is 1 ($p(E|H_1) = 1$). In a dataset taken to be representative of the relevant population, 4 out of 429 individuals also lacked a transverse process on L1; hence, $c_2 = 4$ and $n_2 = 429$, in which c is the count of occurrences within the sample, n is the sample size, and the subscript indicates that these data relate to H_2. The likelihood ratio can, therefore, be calculated as in Equation (2), in which the proportion c/n is used as an estimate of probability. This is the calculation made in [1], which in this instance makes use of a correct formula for the calculation of likelihood ratios.

$$(2) \quad \Lambda = \frac{p(E|H_1)}{p(E|H_2)} = \frac{1}{c_2/n_2} = \frac{1}{4/429} = 107.25$$

In [1], for the illustration of the calculation of likelihood ratios given the second pair of hypotheses stated above (male versus female), a categorical observation is also used:

- E = a forensic anthropologist was of the opinion that the remains were those of a biological male[3]

The method used to reach this opinion was described in [4]. [1] reports that in [4], using a dataset taken to be representative of the relevant population and making opinions based on visual observation of the size of the mastoid, 23 out of 25 male crania were categorized as male ($c_1 = 23$, $n_1 = 25$), and 2 out of 25 female crania were categorized as male ($c_2 = 2, n_2 = 25$). The likelihood ratio can be correctly calculated

[3] Instead of a clear statement of E, [1] used the inexact phrase “estimated sex male”.

as in Equation (3), in which the proportions c/n are used as estimates of probability.[4]

$$(3) \quad \Lambda = \frac{p(E|H_1)}{p(E|H_2)} = \frac{c_1/n_1}{c_2/n_2} = \frac{23/25}{2/25} = 11.5$$

[1], however, does not make the calculation given in Equation (3). Instead, [1] applies its Eq. 4, which (converted into more standard mathematical notation) is given below as Equation (4). In Equation (4), $p(H_1)$ and $p(H_2)$ are the prior probabilities for a missing person to be male and to be female respectively, which, based on published data for counts of missing persons in Canada, [1] gives as 0.58 and 0.42 respectively.

$$(4) \quad \mathrm{K} = \frac{p(E|H_1)}{p(E|H_1)p(H_1)+p(E|H_2)p(H_2)} = \frac{\frac{23}{25}}{\frac{23}{25}\times 0.58+\frac{2}{25}\times 0.42} \approx 1.6$$

Equation (4) is not a correct formula for calculation of a likelihood ratio, and, therefore, K is not a likelihood ratio. Had the numerator of Equation (4) been $p(E|H_1)p(H_1)$ instead of just $p(E|H_1)$, then Equation (4) would have calculated the posterior probability for H_1, i.e., $p(H_1|E)$, see Equation (5).

$$(5) \quad p(H_1|E) = \frac{p(E|H_1)p(H_1)}{p(E|H_1)p(H_1)+p(E|H_2)p(H_2)} = \frac{\frac{23}{25}\times 0.58}{\frac{23}{25}\times 0.58+\frac{2}{25}\times 0.42} \approx 0.94$$

Note, however, that the prior probabilities used in [1] do not necessarily represent the prior probabilities of the trier of fact in the context of a case. The trier of fact could have already heard other testimony that makes their belief about $p(H_1)$ and $p(H_2)$ prior to hearing this testimony different from the published proportions for missing persons in Canada. A posterior probability would not be appropriate for a forensic practitioner to present in the context of a case.

[1] cites [11] as the source of [1]'s Eq. 4, and indeed it is a copy of [11]'s Eq. 3. In

[4] Given the small sample size, a Bayesian approach using beta-binomial models and Jeffreys' reference priors might be preferable ([5]–[10]), but, for brevity, only the simple frequentist approach is presented here.

[11], $p(H_1)$ and $p(H_2)$ were each set to 0.5. The result was Equation (6), which, assuming equal prior probabilities, gives twice the posterior probability for H_1.[5]

(6) $$\mathrm{K}_{p(H_1)=p(H_2)=\frac{1}{2}} = \left.\frac{p(E|H_1)}{p(E|H_1)p(H_1)+p(E|H_2)p(H_2)}\right|_{p(H_1)=p(H_2)=\frac{1}{2}}$$

$$= \frac{p(E|H_1)}{\frac{1}{2}\big(p(E|H_1)+p(E|H_2)\big)}$$

$$= 2\,\frac{p(E|H_1)}{p(E|H_1)+p(E|H_2)}$$

$$= 2p(H_1|E)\big|_{p(H_1)=p(H_2)=\frac{1}{2}}$$

Equation (4) was also used (explicitly or implicitly) in [12] and [13]. This therefore appears to be an error that has been repeated in the forensic anthropology literature over a period of more than two decades.

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